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THE MATHEMATICIAN AND HIS LUCK.[1]

By E. A. Singer.

The mathematician of the seventeenth century is a splendid figure of pride and self-satisfaction, for he believed himself to be the sole possessor of absolute truth. The mathematician of the twentieth century is a much less splendid but much more interesting figure. Assurance has been replaced by skepticism, and satisfaction by discontent. Indeed the modern mathematician is the least happy and least contented of men,—out of sympathy with the universe, and out of humor with his luck. And that not because the gods who put the world together have treated him worse than his fellows. Just the opposite is the case. Of all possible worlds, have they not offered him just that one which is mathematically simplest, which can be described in terms of the easiest arithmetic, the easiest geometry, the easiest kinematics, and a very simple mechanics? The gods have been wonderfully good to the mathematician, and you would think that in this vale of tears he at least would be able to laugh. He is in luck, and it would seem as though he alone were in luck, for in a world as indifferent, ironical, tragic as is this one, the rest of us are not accustomed to find our ease considered, our desires satisfied. Indeed the deepest irony of all lies in the mathematical simplicity and elegance of the machine which the gods have devised for our undoing.

But is the mathematician grateful? Not he. The only lucky man in the world is unhappy just because he is in luck. He does not want his luck. He does not in his heart believe in it. He even goes so far as to suspect that there must be something wrong with himself or with his science or with his way of applying this science, that his position should be so happily anomalous. He would I think welcome any suggestion that would better his condition by making it worse and more human.

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[1] Being an address read after a dinner given by the Philadelphia Section of the Association of Teachers of Mathematics of the Middle States and Maryland.

So at least I have made the mathematician out, and having made him out so, I thought there could be no more delicate and tactful way of thanking him for his hospitality on this occasion than by doing what I can to relieve him of his luck.

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The dying eighteenth century brought forth in heavy travail many new and troubling things, but none newer and more upsetting than that philosophy of Kant which Moses Mendelssohn called the all-disturbing. "A Copernican change of standpoint," was one of the expressions which Kant himself found for the revolution which had taken place within him, and how apt the phrase may be judged from a single passage of the *Kritik der reinen Vernunft* into which Kant has put all his meaning and all his daring,—" The order and uniformity of the phenomena we call nature we ourselves have put there and never should have found them had we not first put them there." The immediate followers of Kant developed this thought of the master by working out more completely the theory of *how* we put law into nature. They saw as Kant had not always seen that the imposing of such laws must be an act of the will, the expression of our choice and freedom. So that experience which had been looked upon by the English school as a mere passive reception of facts, came to be regarded as a process of construction, a work of the creative imagination. " Every fact results from an act," was Fichte's way of putting the result, and with this result whether right or wrong all later philosophy has had to reckon. Its bearing on our particular problem is obvious enough. There is no other way of freeing the mathematician from the curse of his luck than that of assuring him that he made it himself. It is this idea that I venture to put before you.

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The point of departure for the idealistic philosophy of nature was the insight that nature and science are no more capable of independent definition than are right and left, above and below, husband and wife. Nature was still the object of scientific knowledge, but only in the sense that a goal or ideal may be the object of endeavor. Solomon Maimon suggested the figure, ***limit*** *of a series.* As the square root of two is defined by the terms of the series that approaches it, so is nature defined by the series of ever-broadening but always finite insights which make up science in its development. Nature, we may say, is that image which science approaches as the error of observation approaches zero. So viewed, nature is no *Ding an sich* but the name of a certain ideal. Nature is completed science. Science is nature in the making.

If this analysis of meanings is accepted, one outcome is obvious,—the maker of science must be the maker of nature, and if the making of science is no mere passive receiving of hard facts, if on the contrary this making is an act, an act of the creative imagination, then surely it should not surprise us that at least some aspects of the thing made should be after our hearts' desire. This all follows *if* the scientist is permitted to use his imagination, *if* he is allowed to exercise choice. But is he? I can't help thinking that those who maintain the contrary have not lived in laboratories or observatories. They have only imagined what it would be like. They have imagined in the simplicity of their hearts that the experimenter could leave all to his experiment and do nothing himself. It would be beautiful if it were so, but no one knows better than the experimenter that his task is far from being as simple as all this comes to. He finds himself forced to choose between equally permissible images of what is going on around him. Every *laboratorius* knows this, but not every one realizes the range of choice that is open to him. He accepts many choices that have already been made as though the issue were dead. It is to the revival of these issues that a historian of the exact sciences, a Mach let us say, contributes. Moreover the statement of all the alternative images between which choice must be made, is possible only in so far as these images have actually been constructed. It is to this construction that a mathematician of the order of Poincaré contributes.

For these reasons the philosopher attaches great importance to studies which the experimental scientist is likely to view with impatience, namely the history or perhaps I should say the psychology of the sciences called exact and the activities of the mathematician constructing symbolisms that have no immediate application. For these studies contribute more than any other to the emancipation of the scientist by actually making it possible to shatter the scheme of things and to remold it. They open the way to re-creations of nature.

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Of the choices which reflection on the history and psychology of science has brought clearly to the consciousness of the scientist, there are two which are particularly important. The first is the choice between images that are analytically connected, so that we may pass from one to the other by a recognized transformation. Such is the relation between the Ptolemaic and the Copernican pictures of the solar system. The achievement of Copernicus involved no new observation. It was truly the work of the creative imagination.

But how elementary is the transformation of coördinates within the Euclidean system when compared with the transformation of the system itself into one of the non-Euclidean orders. Yet the advantages and disadvantages of such transformation have to be exhaustively considered before we can close the issue between them. Very recently too we have come to consider transformations that involve the relative motion of various systems of coördinates. One may consider what simplicity has been introduced into our description of the phenomena of light by formulating the principle of relativity. May we not judge that we are only beginning to realize the scope and freedom of choices of this order, only beginning to understand our own omnipotence?

In the choices so far considered the alternative images are analytically connected. But there is another order of choice always forced upon the scientist that is something more than a transformation. It is the choice between the infinity of laws that fall within the probable error of his scientific data. If Copernicus illustrates the importance of transformation, Newton gives an example of a choice within limits, for surely the essential part of Newton's contribution was his ability to formulate a single law not identical with, but close enough to the separate inductions of Galileo and Kepler. The contribution of Newton as well as that of Copernicus was an act of imagination. Of course choice between limits raises an experimental issue. Between Newton on the one hand and Kepler and Galileo on the other, the choice is no longer open. But evidently as long as experimental error attaches to our data, the disappearance of an old issue is accompanied by the appearance of a new. It is for this reason that we can always treat the apparently complex as the really simple by the assumption of concealed motions. Nor can this possibility ever be eliminated. Though the range within which our choice may be exercised were reduced to the size of a pin point, this would still be a whole universe of freedom, freedom to suppose what relations we choose between the mathematical points within the pin point. To this is due the development of those images of the concealed which we so crudely and inadequately lump together under the caption of atomic theories.

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I have tried in this brief and sketchy way to suggest the sense in which the history of science is a history not merely of passive observation but also of creative imagination. We can see in what sense it is true that the heavens reflect the glory of Kepler and Newton. And can we not see as well why the mathematician is so perfectly accommodated in a world which he himself has made? I should like, if time permitted, to go further, oh, much further. I should like to raise another question, one that I dare say every one of you is asking himself at this moment,—Why, if the mathematician has made his luck, has he not made more of it? Why isn't all science as simple as geometry? Perhaps I should answer by another question, —Why since Wordsworth made such beautiful poems did he not make more of them and make them more beautiful? Perhaps because it isn't easy to create, perhaps because it is even harder to make a world than it is Columbus-like to find one already made.

But I should like to have made this vague answer more definite. I should like to have suggested that our nature-building proceeds in a certain order. It is not unlike the old fifteen puzzle, which most of you will remember. Nothing was easier than to get the first few numerals in order. Then the thing grew harder and harder, until at the very end you generally decided that it would be better to start all over again. I mean to suggest that in the making of science all that is resistant, complex, unlovely, is left to the late-comer, and that perhaps the way of correcting this will involve beginning all over again. In which case it may well enough turn out that the geometer's ease is just the disease from which the superimposed sciences suffer, that the mathematician's luck is by way of being a human misfortune, or rather an all-too-human mistake. But to follow these hints would be to turn after-dinner into before-breakfast. Perhaps though I have said enough to suggest these two reflections,—first that in so far as the world seems made for the mathematician's ease, he has made it so himself; secondly that we are likely to need his genius to help us in making it over again. I leave him then in the hope that I have lifted a load from his shoulders. Let him not quarrel with his luck, for what is good in it isn't luck, and what is luck and accident isn't good.

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